

## Berg-Koppelaar Quadrature

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Expected payoff in a Black & Scholes world:

$$\int_0^{\infty} P(S_t) f(S_t) dS_t \approx \sum_{i=1}^N w_i f(S_i)$$

Lognormal pdf of an underlying:

$$P(S_t) = \frac{1}{S_t \sigma \sqrt{2\pi t}} \exp \frac{-(\ln S_t/S_0 - [\mu - \frac{1}{2}\sigma^2]t)^2}{2\sigma^2 t}$$

The Berg-Koppelaar quadrature is based on the use of an optimally chosen polynomial to approximate the expected payoff -the theoretical value- of a derivative in a Black & Scholes world. It has an error of order  $2n$ , and is exact for function  $f(x)$  that are polynomials order order  $2n-1$ .

### Symbol list:

$f(x)$	The function that is integrated
$w_i$	Weights
$x_i$	Point (abscissae)
$n$	Number of points