

## Expected value of a geometric Brownian motion raised to a power for $S < K$

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$$E[S_T^p | S_T < K] = S_T^p \exp\left(p\left(Y + (p-1)\frac{1}{2}\sigma^2\right)T\right) N(-d_p)$$

$$d_p = \frac{\ln(S_T/K) + \left(Y - (p - \frac{1}{2})\sigma^2\right)T}{\sigma\sqrt{T}}$$

This equation calculates the expected value of a geometric Brownian motion raised to the power of  $p$  of the part of the geometric Brownian motion that is below  $K$  at time  $T$ . This equation is used in "Pricing and Hedging Power Options" by Ronald C. Heynen and Harry M. Kat.

### Symbol list:

$E[A   B]$	The expectation of $A$ under the condition of $B$
$S_T$	The value of an asset at time $T$
$S_T^p$	The value of an asset at time $T$ raised to the power of $p$
$K$	Strike
$Y$	Yield of the asset. For stocks $Y$ =interest rate, for futures $Y$ =0
$\sigma$	Volatility of the asset
$N(\cdot)$	Cumulative normal distribution function