

Spread option using Gaussian quadrature

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$$C_{SP} \approx \sum_i w_i n(x_i) C_{BS}(S', X', \sigma', Y_1, t)$$

$$S' = S_1 \exp\left(\rho\sigma_1\sqrt{t}x_i - \frac{1}{2}\sigma_1^2\rho^2t\right)$$

$$X' = S_2 \exp\left(\sigma_2\sqrt{t}x_i + (Y_2 - \frac{1}{2}\sigma_2^2)t\right) + X$$

$$\sigma' = \sigma_1\sqrt{1 - \rho^2}$$

This equation uses the Gauss-Legendre quadrature to approximate the value of a spread option. The Gauss-Legendre quadrature abscissas (x_i) are rescaled in the range -4 to +4. The equation is unbiased and gives very accurate results, typical 6 digit accuracy with 16 quadrature points. The method was describes by K. Ravindran in his paper "Low-fat spreads" (1993) RISK 6 (10) 56--57.

Symbol list:

C_{SP}	Price of the spread call option with payoff $\max(S_1 - S_2 - X, 0)$
C_{BS}	Price of a Black & Scholes vanilla option
w_i	Gauss-Legendre quadrature weights
x_i	Gauss-Legendre quadrature abscissas in the range -4 to +4
S_1	Value of the long underlying
σ_1	Volatility of the long underlying
Y_1	Yield of the long underlying
S_2	Value of the short underlying
σ_2	Volatility of the short underlying
Y_2	Yield of the short underlying
ρ	Correlation between the underlyings
X	Strike
t	Time till expiration
$n(x)$	Normal density function