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Binomial Tree, geometric Brownian motion: Tian

created by **Thijs van den Berg**

$$u = \tfrac{1}{2}e^{(Y+\sigma^2)t}\left(e^{\sigma^2 t} + 1 + \sqrt{e^{\sigma^4 t^2} + 2e^{\sigma^2 t} - 3}\right)$$
$$d = \tfrac{1}{2}e^{(Y+\sigma^2)t}\left(e^{\sigma^2 t} + 1 - \sqrt{e^{\sigma^4 t^2} + 2e^{\sigma^2 t} - 3}\right)$$
$$S_u = S.u$$
$$S_d = S.d$$
$$p_u = \frac{e^{Yt} - d}{u - d}$$
$$p_d = 1 - p_u$$

This is the Tian version of the Binomial tree. The Binomial tree is a discretized description of geometric Brownian motion which is often used to describe asset behavior. The structure is a recombining tree where the asset S can move either up or down.

Symbol list:

u Up-factor
d Down-factor
σ Volatility
Y Yield of the underlying, for stocks Y=r (interest rate), futures Y=0, currencies Y=(domestic interest rate-foreign interest rate)
t Timestep
S Present value of the asset
S_u Value of the asset after a up movement
S_d Value of the asset after a down movement
$p_u = P(S_u|S)$ Probability of an up movement
$p_d = P(S_d|S)$ Probability of a down movement