

Generalized Black & Scholes option greeks

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Call option:

$$\begin{aligned} \text{delta} &= \frac{dC}{dS} = e^{(Y-r)T} N(d_1) \\ \text{gamma} &= \frac{d^2C}{dS^2} = e^{(Y-r)T} \frac{n(d_1)}{S\sigma\sqrt{T}} \\ \text{vega} &= \frac{dC}{d\sigma} = e^{(Y-r)T} n(d_1)\sqrt{T} \\ \text{theta} &= \frac{dC}{dT} = (Y-r)Se^{(Y-r)T}N(d_1) \\ &\quad + rKe^{-rT}N(d_2) \\ &\quad + \frac{Se^{(Y-r)T}n(d_1)\sigma}{2\sqrt{T}} \\ \text{rho} &= \frac{dC}{dr} = TK e^{(Y-r)T} N(d_2) \end{aligned}$$

Put option:

$$\begin{aligned} \text{delta} &= \frac{dP}{dS} = e^{(Y-r)T} N(-d_1) \\ \text{gamma} &= \frac{d^2P}{dS^2} = e^{(Y-r)T} \frac{n(d_1)}{S\sigma\sqrt{T}} \\ \text{vega} &= \frac{dP}{d\sigma} = e^{(Y-r)T} n(d_1)\sqrt{T} \\ \text{theta} &= \frac{dP}{dT} = (Y-r)Se^{(Y-r)T}N(-d_1) \\ &\quad - rKe^{-rT}N(-d_2) \\ &\quad + \frac{Se^{(Y-r)T}n(d_1)\sigma}{2\sqrt{T}} \\ \text{rho} &= \frac{dP}{dr} = -TK e^{(Y-r)T} N(-d_2) \end{aligned}$$

The equations are the greeks: delta, gamma, vega, theta, rho of the generalized Black and Scholes option model.

Symbol list:

C	Call price
P	Put price
S	Present value of the underlying
Y	Yield of the underlying contract. Y=r for stock, Y=0 for futures
σ	

	Volatility of the underlying contract
r	Continuous compounded interest rate
T	Time till expiration
K	Strike (exercise) price
$N()$	Cumulative normal distribution function
$n()$	Normal density function
$\ln()$	Natural logarithm (base e)