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Correlation between two points in time of geometric Brownian motion

created by **Thijs van den Berg**

$$\mathrm{Var}(\ln S_t) = \sigma^2 t$$
$$\mathrm{Std}(\ln S_t) = \sigma\sqrt{t}$$

$$\begin{aligned}\mathrm{Cov}(\ln S_t, \ln S_T) &= \min[\mathrm{Var}(\ln S_t), \mathrm{Var}(\ln S_T)] \\ &\underset{\text{when } t<T}{=} \sigma^2 t\end{aligned}$$

$$\begin{aligned}\mathrm{Cor}(\ln S_t, \ln S_T) &= \frac{\mathrm{Cov}(\ln S_t, \ln S_T)}{\mathrm{Std}(\ln S_t)\mathrm{Std}(\ln S_T)} \\ &= \sqrt{\frac{t}{T}}\end{aligned}$$

These equations describe the correlation in time of geometric Brownian motion.

When looking at a two moments in time ("t" and "T") of a specific price-path we can observe two basic facts when comparing the paths between now and "t" and "T" respectively.
* the part up to "t" of the two paths are identical
* the part after "t" is uncorrelated with the part up to "t"
Using these two fact, the results are easy to verify.

Symbol list:

S_t	Value of the geometric Brownian motion at time t
S_T	Value of the geometric Brownian motion at time T (with T>t)
Var	Variance
Std	Standard deviation
Cov	Covariance
$\ln$	Natural logarithm